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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 564

PROCEDURE FOR DETERMINING SPEED AND CLIMBING PERFORMANCE OF AIRSHIPS

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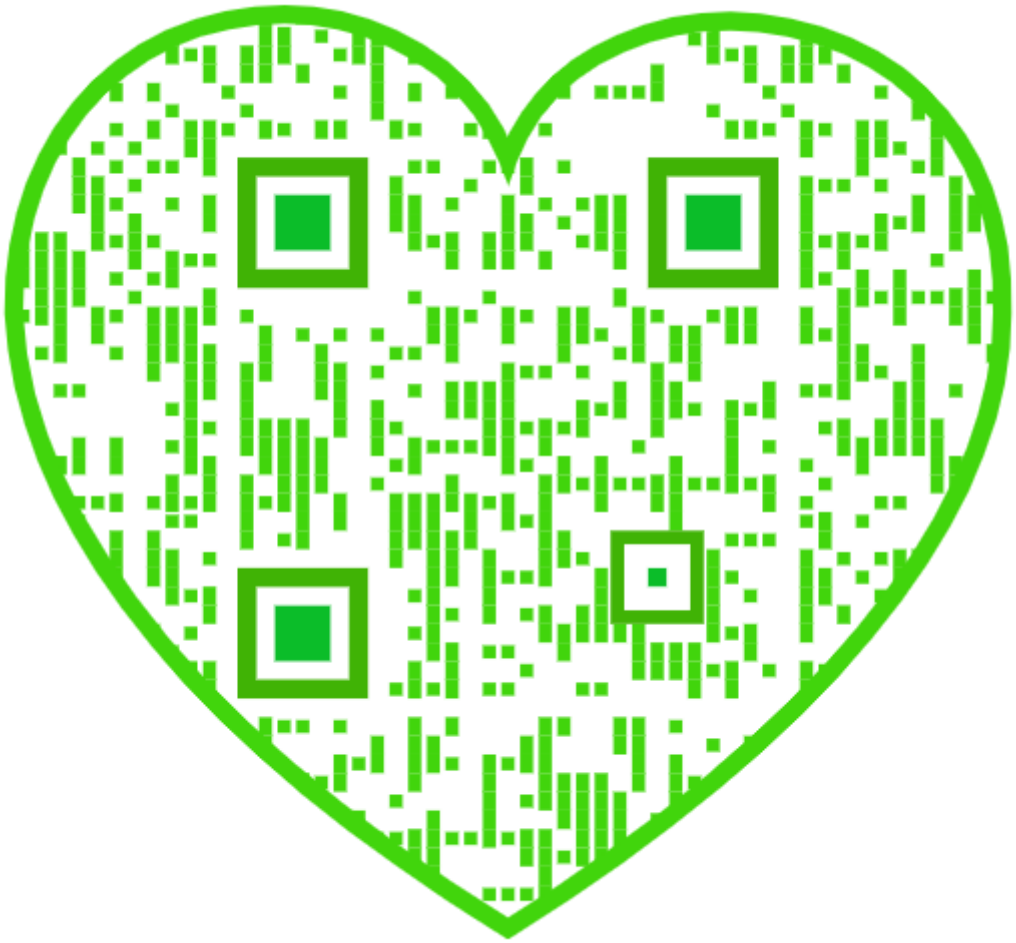

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 564

PROCEDURE FOR DETERMINING SPEED AND CLIMBING PERFORMANCE OF AIRSHIPS

By F. L. Thompson

SUMMARY

The procedure for obtaining air-speed and rate-of-climb measurements in performance tests of airships is described. Two methods of obtaining speed measurements, one by means of instruments in the airship and the other by flight over a measured ground course, are explained. Instruments, their calibrations, necessary correction factors, observations, and calculations are detailed for each method, and also for the rate-of-climb tests.

A method of correction for the effect on density of moist air and a description of other methods of speed-course testing are appended.

INTRODUCTION

The procedure required to obtain accurate measurements of air speed and rate of climb in performance tests of airships is described herein for the instruction and guidance of those who, without having had the benefit of previous experience, are required to conduct such tests. Since it is important that those who conduct the tests should appreciate the necessity of following the correct procedure in all details, the basis for the recommended procedure, as well as an outline of it, is briefly given. The paper is written in an elementary form and the test procedure is outlined in considerable detail so as to minimize as far as possible the necessity for previous knowledge of the factors involved and to avoid the possibility of error in following the correct procedure.

The general methods of measuring the air speed in flight are: by means of instruments attached to the airship and by means of timed flight over a measured course

on the ground. The instrument method may employ one of the numerous types of air-speed head that measure the dynamic pressure, from which instantaneous values of the true air speed can be calculated when the air density is known, or a windmill type of instrument independent of the air density that gives a measure of either the instantaneous or the average true air speed directly, depending on the type of mechanism. Various types of air-speed instruments are described in reference 1. The speed-course method may employ a straight or a triangular course. In either case an average value of true air speed is deduced from the results; the accuracy of the measurements is largely dependent on wind conditions. The procedure outlined herein will be confined to cases in which the instrument method with a pitot-static head is employed and in which the speed-course method with either a straight or a triangular course is used.

Rate-of-climb measurements are made by recording the rate of change of barometric pressure with time, which is then converted to a rate of change of altitude with time in accordance with the change of pressure with altitude for observed air densities.

It will frequently be necessary in describing the procedure for determining air speed and rate of climb to refer to "standard atmosphere," "pressure altitude," and "density altitude." The standard atmosphere is defined in reference 2 and represents approximately average atmospheric conditions as regards relations between true altitude, pressure, temperature, and density. In any actual case there may be a considerable departure from these average conditions. The term "pressure altitude" is the altitude in the standard atmosphere corresponding to an observed barometric pressure; "density altitude" is the altitude in the standard atmosphere corresponding to an observed density. Since altimeters are instruments actuated solely by pressure changes, they can be used to obtain pressure altitude directly. Density altitude can be calculated when the pressure and temperature at a given height are known. For convenience in making the calculations subsequently described, figures 1 and 2, showing the relations between pressure and density in the standard atmosphere, are included. The true altitude, which is of practically no importance in the present case, can be determined accurately only when the pressure at a given height and the temperatures at all altitudes below this height are known. (See reference 3.)

AIR-SPEED MEASUREMENTS

Instrument Method

Effect of velocity field.- The velocity of the air relative to an airship in flight is influenced over a wide field by the presence of the airship and control car or other protuberances. The local velocity at any point is dependent on the shape of the airship and protuberances, on the location of this point relative to the body or bodies causing the disturbance, and on the direction of the relative wind. In addition to this general velocity field, which extends to a great distance in all directions, there is the so-called "boundary layer" of air close to the body in which the velocity is retarded by friction. Although this boundary layer increases in thickness from the bow to the stern, it is relatively thin and easily avoided in making measurements of air speed.

The general nature of the velocity field close to the hull is indicated by the distribution of normal pressure on the hull. At the bow and stern the normal pressures are higher than true static, and the velocities in these regions are correspondingly lower than the true air speed by as much as 100 percent. Amidships the normal pressures are less than true static and the velocities are correspondingly higher than the true air speed by as much as about 10 percent. Between these regions of low and high velocity there are marginal regions in which the true air speed prevails, but they are of small practical significance as regards air-speed measurements. The location of these marginal regions is dependent on the trim of the airship and local irregularities of contour and, furthermore, tests would be required to establish the location for any given trim condition.

In order to avoid the effect of the velocity field, it is necessary to place the air-speed head used in speed trials at a considerable distance from the airship. An indication of the distance that is required is shown in figure 3. The curves shown in this figure apply to calculated values for the U.S.S. Akron hull at zero pitch and, although they do not apply exactly to other airships, they can be regarded as approximately representative of the general case. As shown in the figure, beyond the distance of 1-1/2 diameters from the midship section the error in local velocity becomes very small. Thus with an

airship such as the TC-13 (maximum diameter of 54 feet), a suspension length of 75 to 100 feet, which is a practicable length, seems to insure satisfactory results. It should be noted that the actual distance from the airship to the suspended instrument will be appreciably reduced by a curvature of the cable in flight.

Pitot-static head.- Theoretically the pitot-static head has two openings, one of which is normal to the air stream and is subjected to the total or impact pressure P caused by bringing the air to rest, whereas the other opening is parallel to the air stream and is subjected to the static pressure p. The relation between these two pressures is $P = p + \frac{1}{2} \rho V^2$ where ρ is the mass density of the air and V is the true air speed. The two openings in the pitot-static head are connected to a pressure gage that records the difference between these pressures, $\frac{1}{2} \rho V^2 = q$, which is defined as the dynamic pressure. This ideal condition is seldom exactly realized, owing to the structural details of the head itself, so that the recorded pressure is actually an erroneous value q', which for practical purposes can be regarded as proportional to q regardless of the speed. It is therefore necessary to calibrate the pitot-static head after it is constructed in order to establish the correction factor $K = q/q'$. Knowledge of pitot-static heads is sufficient to permit designing a head for which this factor is very close to unity.

There are an infinite number of possible forms for pitot-static head. A satisfactory design based on convenience of use and ruggedness is shown in figures 4 and 5. A straight tube with a rounded nose has an opening in the nose to obtain the total pressure P and openings around its circumference at a distance of 3 diameters from the nose to obtain the static pressure p. The curvature of the nose portion is in accordance with the equation

$$\frac{r}{d} = \frac{4}{3} \sqrt{\frac{x}{d}} - \frac{x}{d} + \frac{1}{6} \left(\frac{x}{d} \right)^2$$

where d is the maximum diameter of the tube and r is the radius of a section at any distance x measured from the extreme tip. The curvature terminates at $x = d$. The tube, which has a diameter of 2 inches, is loaded with lead to make it heavy enough for satisfactory suspension and is equipped with stabilizing surfaces to keep it point-

ed in the direction of flight. The head weighs about 16 pounds, this weight being necessary in order to reduce the tendency for the suspension cable to swing back owing to its drag.

Method of suspension.- The pitot-static head is suspended on a small flexible cable (1/8-inch diameter is satisfactory) and the static and dynamic pressures are conducted through a pair of rubber tubes. The tubes and cable are tightly encased in a longitudinal strip of adhesive tape having a width considerably greater than the circumference of the encased tubes and cable so that a generous overlap of the edges of the tape is obtained. This type of suspension replaces the single-duct cable shown in figure 5, which is used when only the static pressure is to be measured. In order to avoid excessive drag of the tubing, the outside diameter of these tubes should not exceed 1/4 inch and, owing to the possibility of lag in the transmission of varying pressures through these tubes, they should have an internal diameter of at least 3/32 inch. The recommended size of tubing is 3/16 inch with a wall thickness of 1/32 inch, which gives an outside diameter of 1/4 inch.

Pressure gage or manometer.- The pressure difference at the ends of the tube can be observed by means of a commercial type of air-speed indicator, at least for the upper end of the speed range, or by means of a liquid manometer. The air-speed meter should be checked for leaks and to determine whether its calibration is affected by temperature or position error (effect of changes in the direction of the gravitational force with respect to the instrument) and whether there is any hysteresis. A liquid manometer should be so designed that it is not materially influenced by changes in attitude by providing it with two reservoirs symmetrically placed on either side of the glass tube in which the height of the liquid is observed. The error due to deviation of the manometer attitude from the vertical will then be represented by the deviation of the cosine of that angle from unity. The design should be such that surging of the liquid from one reservoir to the other does not develop an appreciable suction at the juncture of the tube with the reservoir system, which would tend to lower the reading in the glass tube. Whether or not such an effect exists can readily be detected by observing the manometer reading as the manometer is tilted slowly from side to side. The reservoirs should be large in comparison with the volume of the glass tube so that the change of level

in them is small as compared with that in the glass tube. Such an arrangement tends to provide the maximum sensitivity. The sensitivity is also improved by the use of a relatively light liquid such as alcohol rather than water. The density of alcohol or any other light liquid varies considerably with temperature, however, and alcohol also tends to absorb moisture so that the density may change with time. If alcohol is used, the density must therefore be carefully checked at the time the tests are made. Alcohol is recommended in preference to any other light liquid that might be used in place of water. The manometer must be calibrated.

When air speeds are varying rapidly, as in deceleration tests for which it is necessary to obtain a close relation between air speed and time, it is advisable to use a recording instrument such as the N.A.C.A. recording air-speed meter. This instrument gives a continuous photographic record of the dynamic pressure with a time scale, but requires attention by an operator who is thoroughly familiar with it. For most airship tests visual observations are sufficient.

Lag error due to change of altitude.- One additional point that must be considered when measurements are made while climbing or descending is that erroneous readings may be obtained unless precautions are taken to eliminate lag effects in the pressure lines. Owing to the change of static pressure with height in this case, there is a change of static pressure with time. One side of the air-speed system is subjected simply to the static pressure p and the other side to the total pressure P which is the sum of the static pressure and the dynamic pressure q. Actually p is very large compared with q so that for purposes of this argument the pressure in the two sides of the system can be regarded as approximately equal. Then, since p varies with time, both sides of the system are subjected to the effect of the varying pressure. Owing to the difference in the volume of air in the two sides of the line or to a difference in the restrictions in the lines, the effect of this varying pressure may not be the same in both lines, with the result that the recorded dynamic pressure will be in error. A simple test shows whether or not the lag effects are equal. A small pressure is applied simultaneously at both openings of the pitot-static head and as this pressure is released so as to vary the pressure rapidly at the same rate in each side of the system, the reading of the pressure gage is observed. If the

gage shows an appreciable deflection from zero, the system requires modification. This modification consists simply in adding additional volume at the gage end of the side of the system that shows the least lag, that is, the more rapid drop in pressure. It may only be necessary to add a small length of tubing to provide the additional volume required. For an ordinaty air-speed meter, however, there is a large difference in the volume on the two sides of the gage so that it may be necessary to add a large volume to compensate for this inequality.

Errors due to wind gradient.- It is possible that under certain conditions there may be a sufficient gradient of wind velocity with altitude so that the suspended head and the airship may be traveling at different velocities relative to the air. In order to avoid the possibility of an appreciable error from this source it seems advisable to use the average of readings obtained by flights in opposite directions.

Calculation of air speed from observed data.- From the basic relation $q = \frac{1}{2} \rho V^2$, two expressions are derived, namely,

$$V_i = 45.08 \sqrt{q}$$

and

$$V = V_i \sqrt{\delta}$$

where V_i is the indicated air speed in miles per hour

q, the dynamic pressure in inches of water

V, the true air speed in miles per hour

$\delta = \rho_o/\rho$, the ratio of air density at standard sea-level conditions to the density at which tests are made. At standard sea-level conditions the air is assumed to be dry, the barometric pressure ρ_o is 29.92 inches of mercury, and the temperature T_o is 59° F. The density ρ_o for these conditions is 0.002378. The density ρ for any other condition of temperature and pressure for dry air can be found from the relation

$$\rho = \rho_o \times \frac{p}{P_o} \frac{459.4 + T_o}{459.4 + T}$$

which, upon substitution of the above-mentioned standard values of temperature and pressure, reduced to

$$\rho = 0.04120 \times \frac{p}{459.4 + T}$$

where p is the observed pressure in inches of mercury and T, the observed temperature in degrees Fahrenheit. For the density ratio we can write

$$\delta = \frac{\rho_o}{\rho} = 0.05772 \times \frac{459.4 + T}{p}$$

Moisture in the air reduces the value of ρ slightly and, if the effect of the moisture is neglected, the result is a small negative error in the calculated velocity. This error can generally be neglected, but for extreme precision, humidity should be taken into account as shown in Appendix I. The pressure p in inches of mercury may be found from the observed pressure altitude in feet by reference to standard altitude tables or charts. (See fig. 1.) A more convenient method is to have the calibration of the altimeter used in the tests plotted against pressure in inches of mercury.

The observed data obtained in flight tests cannot be used in the foregoing equations without some initial steps. The first step in any case is to correct the observed readings in accordance with the calibration of the pressure gage used in the tests. The subsequent steps depend upon the type of instrument used and the nature of the calibrations. Two cases are assumed:

a. The dynamic pressure is expressed as q' in terms of the height of a liquid.

b. Dynamic pressure is expressed as V_i' in mile-per-hour units.

For case (a) the next step is to find $q = q' r K$ where $r = \frac{\text{specific weight of liquid}}{\text{specific weight of water}}$ and K is the pitot correction factor. For case (b) the next step is to find $V_i = V_i' \sqrt{K}$.

Speed-Course Method

Flight observations.- Measurements of true air speed can be obtained by flying over a straight speed course in opposite directions or over a triangular course. (See also Appendix II for other methods.) The deduction of true air speed from the results of such tests presupposes that the course is closely followed, that the wind speed is constant as regards both its magnitude and direction, and that the timing is accurate. For satisfactory results the wind should be steady and of low velocity relative to the speed of the aircraft. Large cross-wind components are likely to introduce difficulty in following the required ground course. Accurate timing demands care in determining the exact instant a specified point is passed. The observer's line of sight should be directed normal to the flight path and, in order to insure accuracy, the landmark should be a line at right angles to the direction of flight or two points on such a line.

Calculations.- If a straight course is used the proper method of evaluation is to find

$$V_a = \frac{\frac{S}{t_1} + \frac{S}{t_2}}{2} \times \frac{1}{1.467}$$

where S is the length of the course in feet

t_1 and t_2, the times in seconds for runs in opposite directions

V_a, the true air speed in miles per hour uncorrected for the effect of a cross wind

In general, it is not desirable to attempt to fly such a course unless the wind is approximately parallel to the course but, if there is an appreciable cross-wind component, a correction can be made by the most convenient of the two following methods:

$$V = \frac{V_a}{\cos \alpha}$$

or

$$V = \sqrt{V_a^2 + (V_w \sin \theta)^2}$$

where V is the true air speed in miles per hour

α, the angle of drift

V_w, the wind speed in miles per hour

and θ, the angle between the direction of the wind and the speed course

If the triangular course is used, the true air speed can best be determined graphically, the analytical solution being too laborious and inconvenient for ordinary use. One point that should be mentioned in this connection is that the average of the ground speeds for the three legs does not give the correct result. The error is dependent on the shape of the triangle, the magnitude of the wind velocity as a percentage of the speed of the aircraft, the direction of the wind relative to the orientation of the triangle, and, unless the triangle is equilateral, on the direction of flight around the course.

The graphical solution is illustrated in figure 6. The geographical orientation of the three legs of the triangle is required. Vectors representing the ground speeds V_1, V_2, and V_3 along each of these three legs from a common point x are laid out in directions corresponding to the orientation of the appropriate legs. The extremities A, B, and C of these vectors determine a circle, the center O of which can be found by a geometrical construction. This construction consists simply of finding the mutual intersection of the perpendicular bisectors of the three sides of the triangle A, B, and C. The radius of the circle represents the true air speed V and a vector drawn from X to O represents the magnitude and direction of the wind speed V_w. If drift angles were observed during the flights over the speed course, an indication of the steadiness of the wind can now be obtained by drawing air-speed vectors OA, OB, and OC and comparing the drift angles thus indicated with those observed.

It may sometimes be necessary to interpret data obtained in runs during which the engine speed was not maintained constant for the three legs of the triangle. A fairly satisfactory correction may be possible in such a case; for example, suppose that the engine speed is held constant for two legs of the triangle but is reduced for the third leg. The average speed deduced from the vector diagram will lie between that corresponding to the two en-

gine speeds. The air-speed meter readings for the three legs, even though they are considerably in error, can be used to establish an approximate correction factor, by means of which the air speed corresponding to the higher (or the lower, if desired) engine speed can be found. If, from the air-speed meter readings for the three legs, it is deduced that the air speeds were roughly V_{a_1}, V_{a_2}, and V_{a_3}, then, since $V_{a_1} = V_{a_2}$, the correction factor by which the average air speed deduced from the vector diagram must be multiplied is

$$\frac{3V_{a_1}}{2V_{a_1} + V_{a_3}}$$

If the air speed corresponding to a lower engine speed is desired, the factor becomes

$$\frac{3V_{a_3}}{2V_{a_1} + V_{a_3}}$$

After V has been found, the correct indicated air speed may be calculated from the relation $V_i = \frac{V}{\sqrt{\delta}}$ where δ is the density ratio as previously defined.

Condition of Airship for Speed Trials

Trim or pitch angle (defined as the inclination of the longitudinal axis to the flight path) is an important consideration in speed trials. The drag of the airship increases to a marked extent with increase in pitch angle and, in order to obtain maximum speed, the airship must be at approximately zero pitch. In order to illustrate the effect of pitch, figure 7, which applies to a model of the U.S.S. Akron hull (reference 4), is shown. A pitch angle of 3° causes an increase in drag and power required of 9 percent with elevator neutral and 25 percent with elevator deflected to overcome the pitching moment of the hull; whereas for 6° pitch the increase is 33 percent or 71 percent, depending upon whether or not the elevators are deflected to obtain balance. The angle of pitch equals the inclination of the hull in level flight, and hence can readily be observed in speed trials. In any speed trials,

conditions of heaviness or lightness, the average pitch angle for different speeds, and any other items that might influence the speed, such as unusual protuberances of any sort, modifications of protuberances, condition of engine, etc., should be noted.

It is apparent from the foregoing remarks that if, in normal operation, the airship is to be flown heavy, it would be advisable to determine the speed characteristics for the range of loads likely to be carried by dynamic lift. The effect of the heaviness on the engine speed and fuel consumption required to fly at a given air speed is likely to be very marked, particularly at low cruising speeds with correspondingly large pitch angles.

Interpretation of Speed Data

It is advisable to plot readings of the air-speed meter of the airship against correct indicated air speed V_i in order to obtain a calibration curve for the complete air-speed installation. This curve will show the combined effect of all errors, the principal one probably being that due to the location of the fixed air-speed head. The magnitude of this error is likely to be dependent to some extent on the angle of pitch, so that the curve thus obtained does not necessarily apply to all conditions of trim. Thus, if the curve is obtained with the airship in static buoyant equilibrium, some deviation can be expected when the airship is heavy or light, and also when it is turning.

True speed V should be plotted against correct engine speed N. This curve has important characteristics. If the airship is in static buoyant equilibrium so that it flies at zero pitch, and hence constant drag coefficient at all speeds, and if there is but one propeller or multiple propellers synchronized to act as one propeller at all speeds, this curve will be very close to a straight line passing through zero. The slope will depend on the drag coefficient of the airship and the propeller characteristics but will be independent of altitude or mechanical condition of the engine. The maximum air speed will depend, of course, on the maximum engine speed obtainable and hence on the altitude and mechanical condition of the engine, but the slope will remain constant. The slope of the curve will be altered if there are alterations to the airship that change either the propeller characteristics or the drag coefficient.

If the airship is not in equilibrium during the speed trials, the curve of V against N should show a varying slope with the ratio of V/N increasing with increasing speed. This type of variation will occur because of the reduction in pitch angle, and hence drag coefficient, with increasing speed. If the plot of the speed results shows this type of variation, it serves, therefore, as an indication that the airship was not in equilibrium and that the measured speeds at the various engine speeds were not as great as could have been obtained with zero pitch. The curve will approach the curve corresponding to zero pitch at high speed.

Summary of Test Procedure for Speed Trials

I. Suspended-head method:

A. Observations:

1. Dynamic pressure from suspended head.

2. Air speed from air-speed meter of airship.

3. Outside air temperature.

4. Inside temperature in control car (unless water manometer is used so that change of density of manometer liquid with temperature is negligible).

5. Pressure altitude (altimeter reading).

6. Inclination.

7. Engine speeds (tachometer readings).

8. Make note of such things as the buoyant condition of the airship, protuberances or openings, behavior of synchronization of the engines, and air conditions that are likely to have bearing on the speed results. It might also be of assistance in interpreting results, to obtain records of control position and to observe the compass readings at short regular intervals.

B. Calculations:

1. Correct readings of dynamic pressure for calibration of instrument to obtain q' (or V_i' if air-speed meter is used).

2. (a) Liquid manometer.- Multiply q' by pitot correction factor K and manometer liquid-density factor r to obtain q, and then find correct indicated air speed from $V_i = 45.08 \sqrt{q}$.

   (b) Air-speed meter.- Find correct indicated air speed V_i from

$$V_i = V_i' \sqrt{K}$$

3. Correct altimeter readings in accordance with calibration to obtain correct barometric pressure p.

4. Same for thermometer to obtain T.

5. Calculate δ from relation

$$\delta = 0.05772 \times \frac{459.4 + T}{p} .$$

6. Calculate true air speed from relation

$$V = V_i \sqrt{\delta}$$

7. Correct tachometer readings in accordance with calibration to find correct engine speed.

C. Plot:

1. True air speed V against engine speed.

2. Correct indicated air speed V_i against reading of air-speed meter of airship.

II. Speed-course method - straight course:

A. Observations:

1. Time to traverse course in opposite directions.

2. Angle of drift α or magnitude V_w and direction θ of wind relative to course.

3. Items 2, 3, 5, 6, 7, and 8 of IA.

B. Calculations:

1. Find V_a from relation

$$V_a = \frac{\frac{S}{t_1} + \frac{S}{t_2}}{2} \times \frac{1}{1.467}$$

2. Correct V_a for effect of cross wind to find true air speed V from

$$V = \frac{V_a}{\cos \alpha}$$

or

$$V = \sqrt{V_a^2 + (V_w \sin \theta)^2}$$

3. Items 3, 4, and 5 of IB to find δ.

4. Calculate correct indicated air speed V_i from $V_i = \frac{V}{\sqrt{\delta}}$

5. Item 8 of IB to find correct engine speed.

C. Plots:

Same as IC.

III. Speed-course method - triangular course:

A. Observations:

1. Time for each leg.

2. Items 2, 3, 5, 6, 7, and 8 of IA.

B. Calculations:

1. Find average true air speed by graphical method (fig. 6).

2. Items 3, 4, and 5 of IB to find δ.

3. Find correct indicated air speed V_i from

$$V_i = \frac{V}{\sqrt{\delta}} .$$

4. Item 7 of IB to find correct engine speed.

C. Plots:

Same as IC.

RATE-OF-CLIMB MEASUREMENTS

General Method

Under average atmospheric conditions represented by the standard atmosphere there is a definite pressure, temperature, and density corresponding to any given altitude. In actual cases there is some departure from the average so that the relations that hold for the standard conditions can be regarded as only approximate for any given case. Altimeters, which are actuated by pressure changes, are graduated in feet in accordance with the variation of pressure with altitude in the standard atmosphere. Hence the reading of an accurate altimeter may be regarded as an exact indication of pressure and an approximate indication of height or true altitude. In general, however, pressure and density, or pressure altitude h_p and density altitude h_d, are the quantities desired. These items can readily be obtained, the former being given directly by

the altimeter reading and the latter being obtained from calculations based on readings of the altimeter and the thermometer.

Although the true altitude may not be known, the true rate of climb can readily be obtained by utilizing the basic relation between altitude change and pressure change.

$$\Delta h = - \Delta p \frac{1}{g\rho}$$

where Δh is an increment of altitude

Δp, the corresponding increment in pressure

ρ, the average air density for the altitude increment being considered

g, the acceleration of gravity ($g = 32.17$ ft./sec.^{2}).

Thus, the altitude change corresponding to a given pressure change depends on the average density ρ, which is determined by the average pressure and temperature for the increment. From the preceding calculation there is obtained for the true rate of climb V_c, the expression

$$V_c = \frac{\Delta h}{\Delta t} = \frac{- \Delta p}{\Delta t} \frac{1}{g\rho}$$

Δt being the time interval required for the observed pressure change Δp. Then the angle of climb is obtained from

$$\theta = \sin^{-1} \frac{V_c}{V}$$

where θ is the angle of climb

V, the true air speed

The units of velocity, time, height, etc., must be consistent as explained later under Calculations.

It now becomes necessary to consider which of the various items are finally desired. An analogy with heavier-than-air craft offers little assistance, since the airship is essentially sustained by static buoyancy rather than by

dynamic forces. Ceiling is determined by volumes and weights in relation to density, rather than by engine power, and is a height which it is not safe to exceed rather than one which it is impossible to exceed. The ceiling of an airship can probably be determined better from calculations than from actual tests. Below the ceiling no more power is required to climb than to fly level as long as no dynamic lift is required. The factors that limit the ability to ascend or descend are essentially the pitch control, or maximum inclination permitted by the design, and the capacity for maintaining correct gas pressure when the atmospheric pressure is varying. The altitude at which the ascent or descent is made is generally of no great importance. It appears, therefore, that we are concerned chiefly with the rate at which the pressure varies $\Delta p/\Delta t$, or the equivalent rate of change of pressure altitude $\Delta h_p/\Delta t$, and the angle of climb θ. If climbs were to be made with dynamic lift the climbing ability would tend to become definitely dependent on engine performance, in which case it appears that the true rate of climb V_c should be obtained as a function of altitude, the pressure altitude h_p probably being better for this purpose than density altitude h_d, although there is some doubt as to which should be used.

Instruments

Some instruments indicate rate of climb directly, the reading of the instruments being dependent on the rate of change of pressure. Such instruments are not recommended for test work, although they are useful in determining at a glance whether one is ascending or descending and the approximate rate. The standard types of Kollsman altimeters for airplanes are small compact instruments that are generally satisfactory and can be recommended for climb tests. These instruments have an appreciable friction and must be lightly tapped to insure accuracy of the readings, unless they are vibrated by other means. When thus vibrated there should be no perceptible hysteresis.

The altitude scale is divided into feet in accordance with the change of pressure with altitude in the standard atmosphere. The instrument, of course, should be calibrated before it is used for test purposes, and it may be convenient to have the calibration show the scale reading in feet against pressure in inches of mercury. The Kolls-

man instrument, like most other altimeters, has an adjustable zero but, in order that one calibration shall suffice, it is desirable that this adjustment be locked in position before the calibration is made. The instrument then becomes essentially a simple aneroid barometer reading barometric pressures in foot units.

The Kollsman instruments are usually equipped with a fitting on the back of the case that permits the pressure chamber to be connected to a source of true static pressure. For airships this connection can be ignored since at the relatively low speeds obtainable with airships the pressure in the control car will not differ from the true static pressure by an amount sufficient to introduce a serious error into the barometric pressure. It seems probable that the error in pressure altitude from this source would not be more than about 40 feet at a speed of 70 miles per hour, and less at lower speeds. The error will probably depend to some extent upon whether windows are open or closed. If absolute precision were desired, it would be necessary to connect the instrument to a suspended static head.

Any type of calibrated thermometer will probably be satisfactory for determining the free-air temperature if it is freely exposed to the outside air. Some consideration should, however, be given to the lag characteristics of the thermometer because for extreme rates of ascent or descent the lag may introduce an appreciable error. The error for any given type of thermometer will be proportional to the rate of change of temperature and hence, in general, to the rate of change of altitude. The temperature ordinarily varies with altitude at the rate of about 3° F. per thousand feet, so that if the rate of ascent or descent were 2,000 feet per minute the temperature would vary 0.1° F. per second. According to data given in reference 5, the errors in reading with different thermometers for this case would be approximately as follows:

Laboratory thermometer, mercury in glass	1° F.
Laboratory thermometer, liquid in glass	$1\frac{1}{2}^{\circ}$ F.
Strut thermometer, liquid in glass, flat bulb	$1\frac{1}{2}^{\circ}$ F.
Strut thermometer, liquid in metal helical bulb	$1\frac{1}{2}^{\circ}$ F.
Strut thermometer, liquid in glass, large cylindrical bulb	3° F.

Since an error of 1° F. introduces an error of only about 0.2 percent in the calculated density, it appears that the error due to lag can usually be neglected even for the high rate of ascent or descent assumed in this case. If greater precision is desired, however, the relation between temperature and altitude can be established immediately before or after the tests by readings made under steady conditions, or at least while the variation in altitude is slow.

When timing the ascent or descent, the increments of altitude for which the time is taken should not exceed 1,000 feet and smaller increments should be used if feasible. The time for equal increments of altitude change or the altitude change for equal increments of time may be observed, depending upon which method is more convenient with the apparatus available. Probably the best accuracy will be obtained by the former method with two or more stop watches being used so that one watch can be started and the other stopped at each interval, and the time readings noted between intervals. Another satisfactory method is to use a bank of stop watches mounted on a single board and so arranged that they can be simultaneously started and independently stopped, one watch being stopped for each increment of altitude observed.

CALCULATIONS

It will be assumed that the flight observations give observed altimeter readings h_{p_1}', h_{p_2}', etc., and times t_1, t_2, etc., corresponding to those altitudes. Air temperatures T_1, T_2, etc., must also be known, of course, before the rates of climb can be calculated. Furthermore, it is assumed that the air speed is observed so that the true air speed can be found.

The first step is to find the barometric pressures corresponding to the observed altimeter readings. If the calibration of the instrument is plotted against pressure in inches of mercury as previously suggested, pressures p_1, p_2, etc., will be obtained directly from the calibration. If the calibration is plotted only against correct pressure altitude h_p, it will be necessary first to find h_{p_1}, h_{p_2}, etc., and then, by reference to standard altitude tables or charts (see fig. 1) to find p_1, p_2, etc.

The next step is to find the air densities ρ_1, ρ_2, etc., corresponding to the observed pressures and temperatures.

$$\rho_1 = 0.04120 \times \frac{p_1}{459.4 + T_1}$$

where p_1 is in inches of mercury and T_1 is in degrees Fahrenheit. Subsequent values of ρ are calculated in a similar manner. Thus, for the first increment of altitude $\Delta p = p_2 - p_1$. the corresponding increment of time is $\Delta t = t_2 - t_1$, and the rate at which the pressure varies is

$$\frac{\Delta p}{\Delta t} = \frac{p_2 - p_1}{t_2 - t_1}$$

in inches of mercury per second when Δp is in inches of mercury and Δt is in seconds or

$$\frac{\Delta p}{\Delta t} = \frac{p_2 - p_1}{T_2 - T_1} 13.59$$

in inches of water per second.

The average value of ρ is

$$\frac{\rho_1 + \rho_2}{2}$$

Then, with pressure still expressed in inches of mercury, the rate of climb in feet per second is

$$V_c = - 70.7 \frac{(p_2 - p_1)}{t_2 - t_1} \frac{1}{\frac{(\rho_1 + \rho_2)}{2} 32.17}$$

the constant 70.7 being the conversion factor for reducing pressure in inches of mercury to pounds per square foot.

The angle of climb θ is given by

$$\theta = \sin^{-1} \frac{V_c \text{ (f.p.s.)}}{1.467 \text{ V (m.p.h.)}}$$

For example, assume the following data to have been observed in a climb:

Altitude:

$h_{p_1}' = 3{,}000$ feet

$h_{p_2}' = 4{,}000$ feet

Time:

$t_1 = 0$

$t_2 = 62$ seconds

Temperature:

$T_1' = 75^\circ$ F.

$T_2' = 73^\circ$ F.

Indicated air speed:

$V_i' = 50$ miles per hour (average)

After the altimeter calibration has been consulted, which it is assumed has been plotted so as to show both the correct pressure altitude and actual pressure corresponding to a given altimeter reading, suppose it is found that

$h_{p_1} = 3{,}250$ feet

$h_{p_2} = 4{,}475$ feet

the corresponding pressures being

$p_1 = 26.57$ inches mercury

$p_2 = 25.39$ inches mercury

Similarly, from the thermometer and air-speed meter calibration is obtained

$T_1 = 77^\circ$ F.

$T_2 = 75^\circ$ F.

$V_i = 47$ miles per hour

The densities are

$$\rho_1 = 0.04120 \times \frac{26.57}{459.4 + 77} = 0.002040$$

$$\rho_2 = 0.04120 \times \frac{25.39}{459.4 + 75} = 0.001957$$

Then, the rate of change of pressure altitude is

$$\frac{\Delta h_p}{\Delta t} = \frac{4475 - 3250}{62} = 19.75 \text{ f.p.s. (1185 f.p.m.)}$$

and the rate of change of pressure

$$\frac{\Delta p}{\Delta t} = \frac{25.39 - 26.57}{62 - 0} \times 13.59 = -0.259 \text{ in. of water/sec. (-15.54 in. of water/min.)}$$

The true rate of climb

$$V_c = \frac{-70.7\ (25.39 - 26.57)}{62 - 0} \times \frac{1}{\frac{(0.002040 + 0.001957)}{2}\ 32.17}$$

$$= 20.92 \text{ f.p.s. (1255 f.p.m.)}$$

Since the average indicated air speed is 47 miles per hour and the average value of ρ is 0.00200, the true air speed is

$$V = 47 \sqrt{\frac{0.002378}{0.00200}} = 51.2 \text{ m.p.h.}$$

Then, for the angle of climb,

$$\sin \theta = \frac{20.92}{51.2 \times 1.467} = 0.278$$

or

$$\theta = 16.2^\circ$$

Condition of Airship for Climb Tests

The condition of the airship for climb tests may or may not be important, depending upon what factors limit the rate at which an ascent or descent can be made. It does not seem feasible, therefore, to attempt to stipulate in general what the condition of the airship should be. If the ability to maintain proper gas pressure limits the performance, as is usually the case for a descent, the condition of buoyancy and trim would probably be unimportant. If the pitch control limits performance, the condition of buoyancy and trim might be important. For example, assume that the airship is heavy so that a positive pitch angle is required in order to maintain a certain dynamic lift. Aside from the fact that the climbing performance now tends to become dependent on horsepower available, the heaviness will have an important bearing on the pitch control. Airships, in general, are unstable so that a positive angle of pitch introduces a positive pitching moment tending to increase the pitch. A negative trimming moment must be applied. If this negative moment is applied by the elevator the result will be that, in maintaining the positive pitch angle, the average elevator position will be down. The net effect is somewhat as though the neutral position of the elevator were shifted downward so that the range of downward elevator movement is diminished and the range of upward motion correspondingly increased. This change in the effective elevator range may have considerable bearing on the ability to ascend or descend.

Interpretation of Climb Data

The results of the climb tests should show maximum values for angle and rate of ascent and descent. They should also show, insofar as is possible, what characteristics of the airship limit the ability to ascend or descend, as for example, controllability, valve capacity, or blower capacity. In connection with items pertaining to valve and blower capacity, rate of change of atmospheric pressure, as well as the corresponding rate of change of altitude, could well be shown.

Summary of Test Procedure for Climb Tests

A. Observations:

1. Altimeter at regular intervals.

2. Time interval between successive altimeter readings.

3. Outside air temperature corresponding to each altimeter reading.

4. Air-speed meter.

5. Inclinometer.

6. Note amount of elevator control required, condition of static buoyancy, and other items that appear to be significant.

B. Calculations:

(Note: In each of the following equations pressures p are in inches of mercury, times t are in seconds, and temperatures T are in degrees Fahrenheit.)

1. Correct altimeter, thermometer, and air-speed meter readings in accordance with calibration to find correct pressure altitudes h_{p_1}, h_{p_2}, etc., and pressures p_1, p_2, etc.; correct temperatures T_1, T_2, etc.; and correct indicated air speed V_i.

2. Find rate of change of pressure for successive intervals in inches of water per second from

$$\frac{\Delta p}{\Delta t} = \frac{(p_2 - p_1)}{(t_2 - t_1)} \times 13.59.$$

If desired, also find rate of change of pressure altitude in feet per second from

$$\frac{\Delta h_p}{\Delta t} = \frac{(h_{p_2} - h_{p_1})}{(t_2 - t_1)} .$$

3. Find successive values of density ρ_1, ρ_2, etc., from

$$\rho_1 = 0.04120 \times \frac{p_1}{460 + T_1}, \quad \text{etc.}$$

4. Find true rates of climb for successive intervals in feet per second from

$$V_c = \frac{-70.7\ (p_2 - p_1)}{t_2 - t_1} \cdot \frac{1}{\frac{\rho_1 + \rho_2}{2}\ 32.17}$$

5. Find average true air speed V in miles per hour for successive intervals from

$$V = \sqrt{\frac{0.002378}{\left(\frac{\rho_1 + \rho_2}{2}\right)}}\ V_i \quad \text{m.p.h.}$$

6. Find the sine of angle of climb θ in degrees from

$$\sin \theta = \frac{V_c\ (\text{f.p.s.})}{1.467\ V\ (\text{m.p.h.})}$$

and θ from trigonometric tables.

C. Plot:

It is apparent that no plotting is required. If climbs with dynamic lift were made, however, there might be some point in plotting rate of climb against air speed for different amounts of dynamic lift at a given altitude or rate of climb against altitude (preferably pressure altitude) for given amounts of dynamic lift.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 26, 1936.

APPENDIX I

Density of Moist Air

Moist air is slightly lighter than dry air because it is a mixture of air and a small quantity of water vapor (steam), and the latter is lighter than air. The first step required to determine the density of the mixture is to find the partial pressure of the water vapor so that the total barometric pressure p may be divided into two parts:

$$p = p_a + p_w$$

where p_a is the partial pressure of air

and p_w is the partial pressure of the water vapor.

Wet-bulb and dry-bulb temperature readings are required in order to calculate p_w by means of Carrier's equation

$$p_w = p_w' - \frac{(p - p_w')(T - T_w)}{2755 - 1.28\,T_w} \text{ inches of mercury}$$

where T is the dry-bulb temperature in degrees Fahrenheit

T_w, the wet-bulb temperature in degrees Fahrenheit

p_w', vapor pressure in inches of mercury corresponding to the temperature T_w

p, barometric or total pressure in inches of mercury. (See standard textbooks of thermodynamics for a more complete explanation; for example, reference 6.)

In order to find p_w' it is necessary to refer to standard steam tables for saturated steam (reference 7). Table I, which has been copied from reference 7, has been included for convenience. The density ρ_w of the water vapor can then be found from

$$\rho_w = \frac{0.0256\,p_w}{459.4 + T}$$

The density of the dry air is

$$\rho_a = \frac{0.04120\ p_a}{459.4 + T}$$

and the density ρ of the mixture is the sum of the two, or

$$\rho = \frac{0.0256\ p_w + 0.04120\ p_a}{459.4 + T}$$

In order to show the error involved by neglecting the humidity, the ratio of the densities of humid and dry air at any given temperature and pressure can be written

$$\frac{\rho(\text{humid})}{\rho(\text{dry})} = \left(1 - 0.378\ \frac{p_w}{p}\right)$$

Example:

$T = 80^\circ$ F.

$T_w = 70^\circ$ F.

$p = 29.42$ inches of mercury

From table I the vapor pressure p_w corresponding to the wet-bulb temperature of 70° is found to be 0.739 inch of mercury. Then

$$p_w = 0.739 - \frac{(29.42 - 0.739)\ (80 - 70)}{2755 - 1.28 \times 70} = 0.628$$

and

$$p_a = 29.42 - 0.63 = 28.79$$

so that

$$\rho = \frac{(0.0256 \times 0.63) + (0.04120 \times 28.79)}{459.4 + 80} = 0.002228$$

The ratio

$$\frac{\rho(\text{humid})}{\rho(\text{dry})} = 1 - 0.378 \times \frac{0.63}{29.42} = 0.992$$

which shows that the error in density due to neglecting humidity would have been 0.8 percent. The results of this error as regards the conversion of indicated air speed to true air speed would have been an error of -0.4 percent in true air speed.

APPENDIX II

Additional Speed-Course Methods

In addition to the speed-course methods discussed in the main body of the text, Dr. Arnstein of the Goodyear-Zeppelin Corporation has recommended two additional methods. For one of them two neighboring but not necessarily adjoining straight courses arranged in the shape of an L or T are used. The two legs are followed in one direction and then retraced. The evaluation can be made as for the triangular course except that there are four instead of three factors from which to determine the circle, so that to some extent a check on the accuracy is obtained. The other method is limited in its application to parts of the country where long parallel landmarks such as roads are available. When such landmarks are available, the method appears to have considerable advantage over other speed-course methods. A summary of the method as described in a Goodyear-Zeppelin Corporation report by Dr. Klemperer, follows:

From an accurate map two parallel roads, say 5 miles apart, are selected as the parallel landmarks. A compass course is set exactly at right angles to these roads and held without regard to the ground path as the crossing is made from one road to the other. (See fig. 8.) The compass course is then reversed and a return is made. It can readily be proved that, from the two crossing times, the true air speed V in miles per hour is obtained without graphical analysis by means of the equation

$$V = \frac{1}{2}\left(\frac{L}{t_1} + \frac{L}{t_2}\right) \times \frac{1}{1.467}$$

where L is the perpendicular distance in feet between the landmarks and t_1 and t_2 are the crossing times for the two directions of flight.

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TABLE I. RELATION BETWEEN TEMPERATURE AND VAPOR PRESSURES OF SATURATED STEAM*

Temperature	Pressure	Temperature	Pressure
° F.	in. Hg	° F.	in. Hg
32	0.1804	70	0.739
33	.1878	71	.764
34	.1955	72	.790
35	.2034	73	.817
36	.2117	74	.845
37	.2202	75	.873
38	.2290	76	.903
39	.2382	77	.933
40	.2477	78	.964
41	.2575	79	.996
42	.2677	80	1.029
43	.2782	81	1.063
44	.2890	82	1.098
45	.3002	83	1.134
46	.3118	84	1.171
47	.3238	85	1.209
48	.3363	86	1.248
49	.3492	87	1.289
50	.3625	88	1.331
51	.3762	89	1.373
52	.3903	90	1.417
53	.4049	91	1.462
54	.4201	92	1.508
55	.4357	93	1.556
56	.4518	94	1.605
57	.4684	95	1.655
58	.4856	96	1.706
59	.5034	97	1.759
60	.522	98	1.813
61	.541	99	1.869
62	.560	100	1.926
63	.580	101	1.985
64	.601	102	2.045
65	.622	103	2.107
66	.644	104	2.171
67	.667	105	2.236
68	.690	106	2.303
69	.714	107	2.372
		108	2.443
		109	2.515
		110	2.589

*Reference 7.

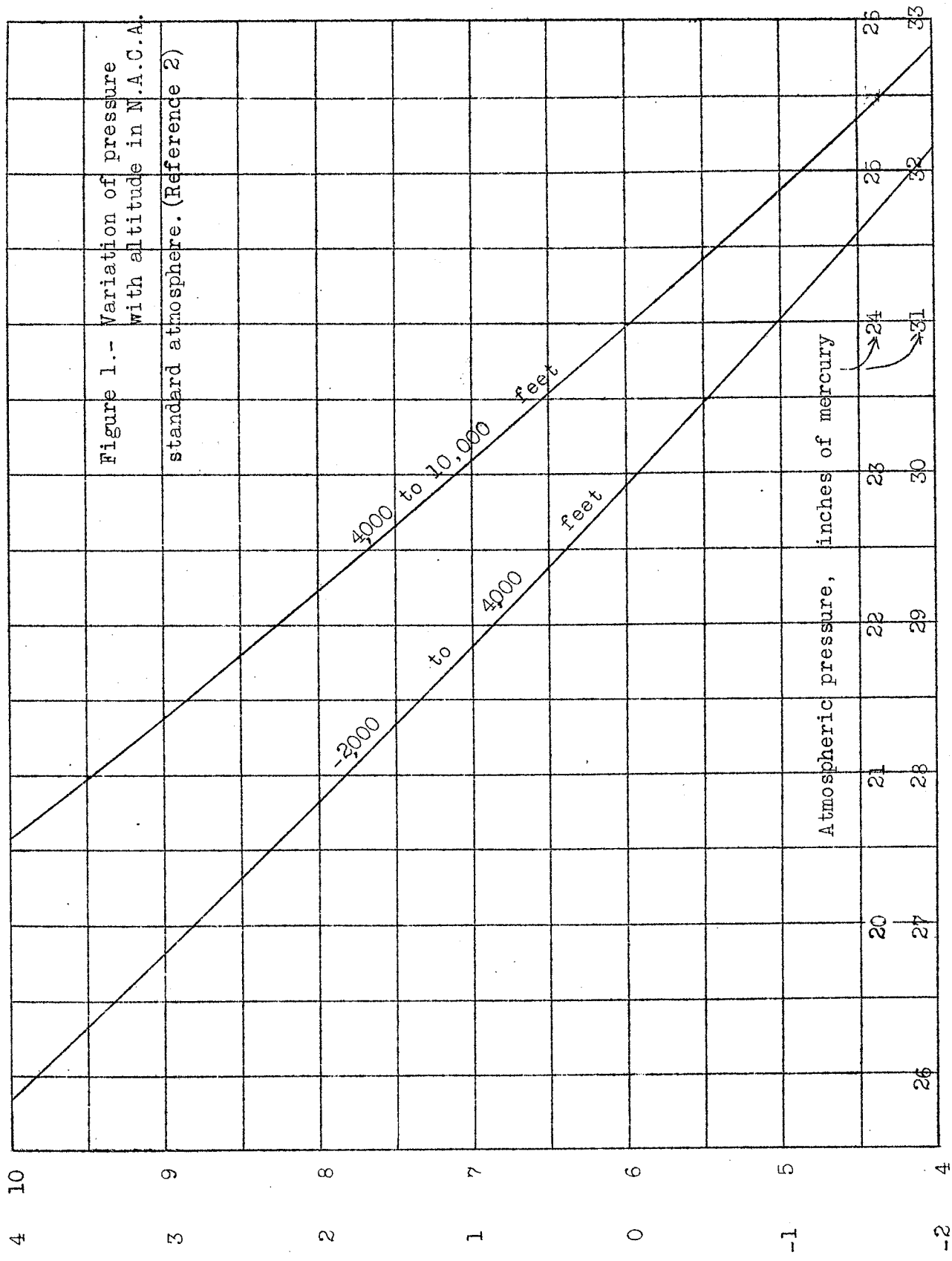


Figure 1.- Variation of pressure with altitude in N.A.C.A. standard atmosphere. (Reference 2)
Atmospheric pressure, inches of mercury
Altitude, thousands of feet
-2000 to 4000 feet
4000 to 10,000 feet

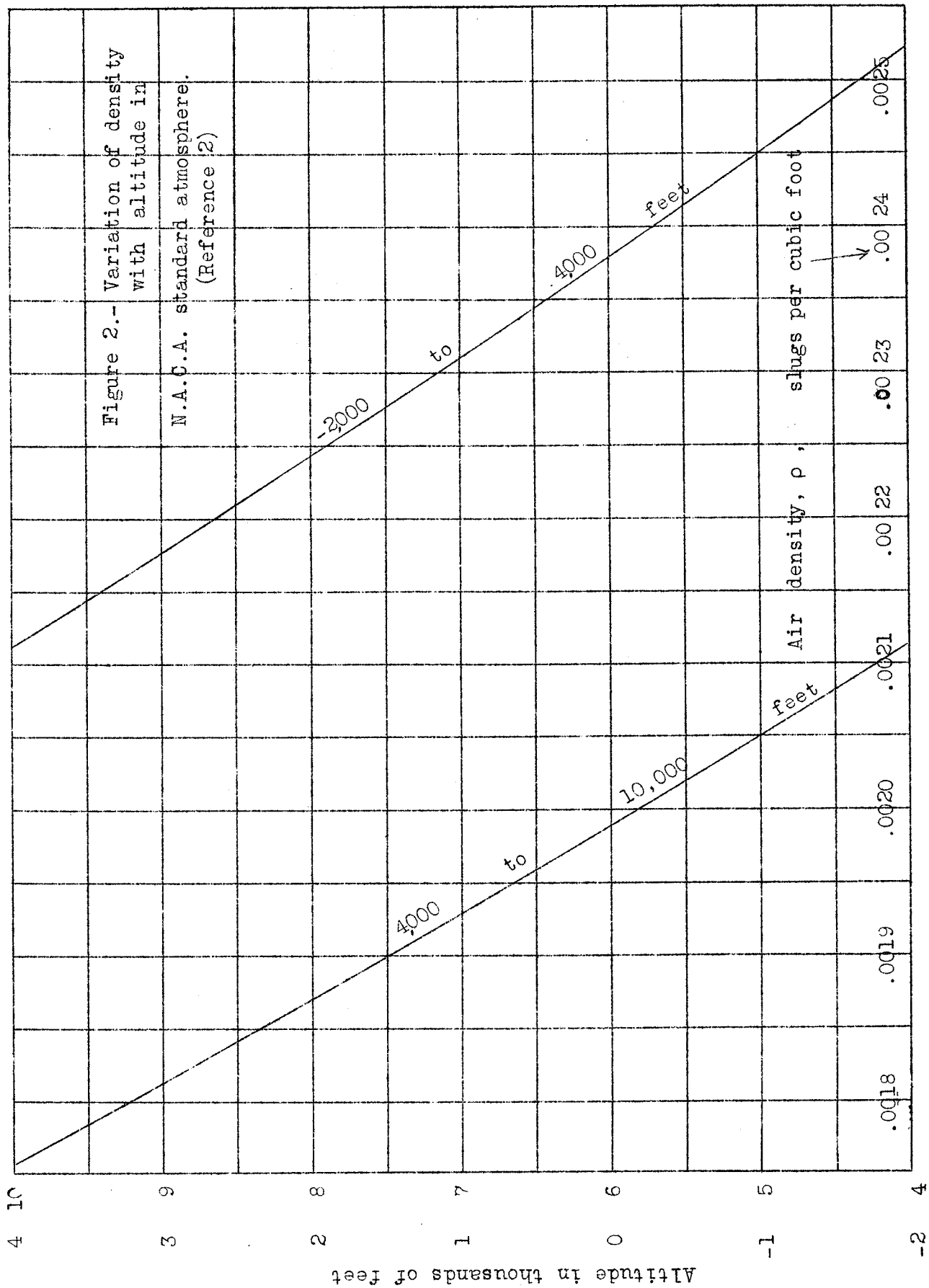
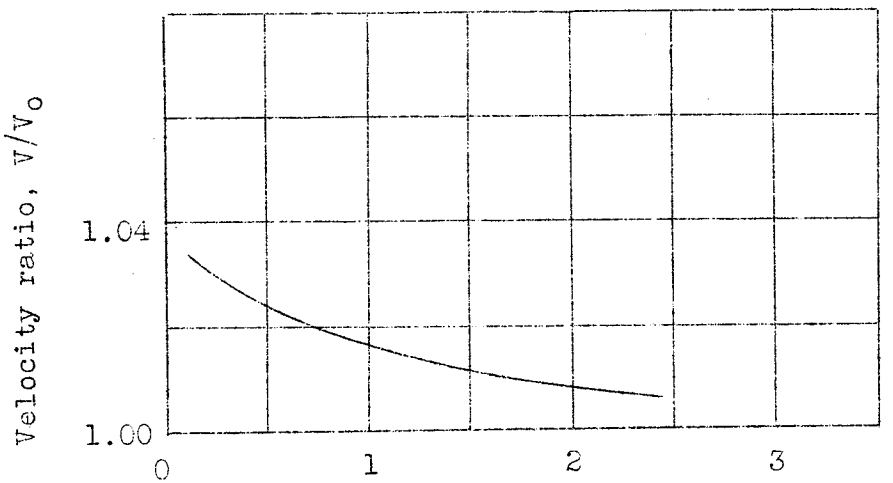


Figure 2.- Variation of density with altitude in N.A.C.A. standard atmosphere. (Reference 2)

V_o , true air speed
V , local air speed
y , distance from hull
D_{max} , maximum diameter



$\frac{y}{D_{max}}$, distance from hull in terms of maximum diameter

Figure 3.- Calculated variation of air velocity with distance from the hull at maximum diameter for the U.S.S. Akron.

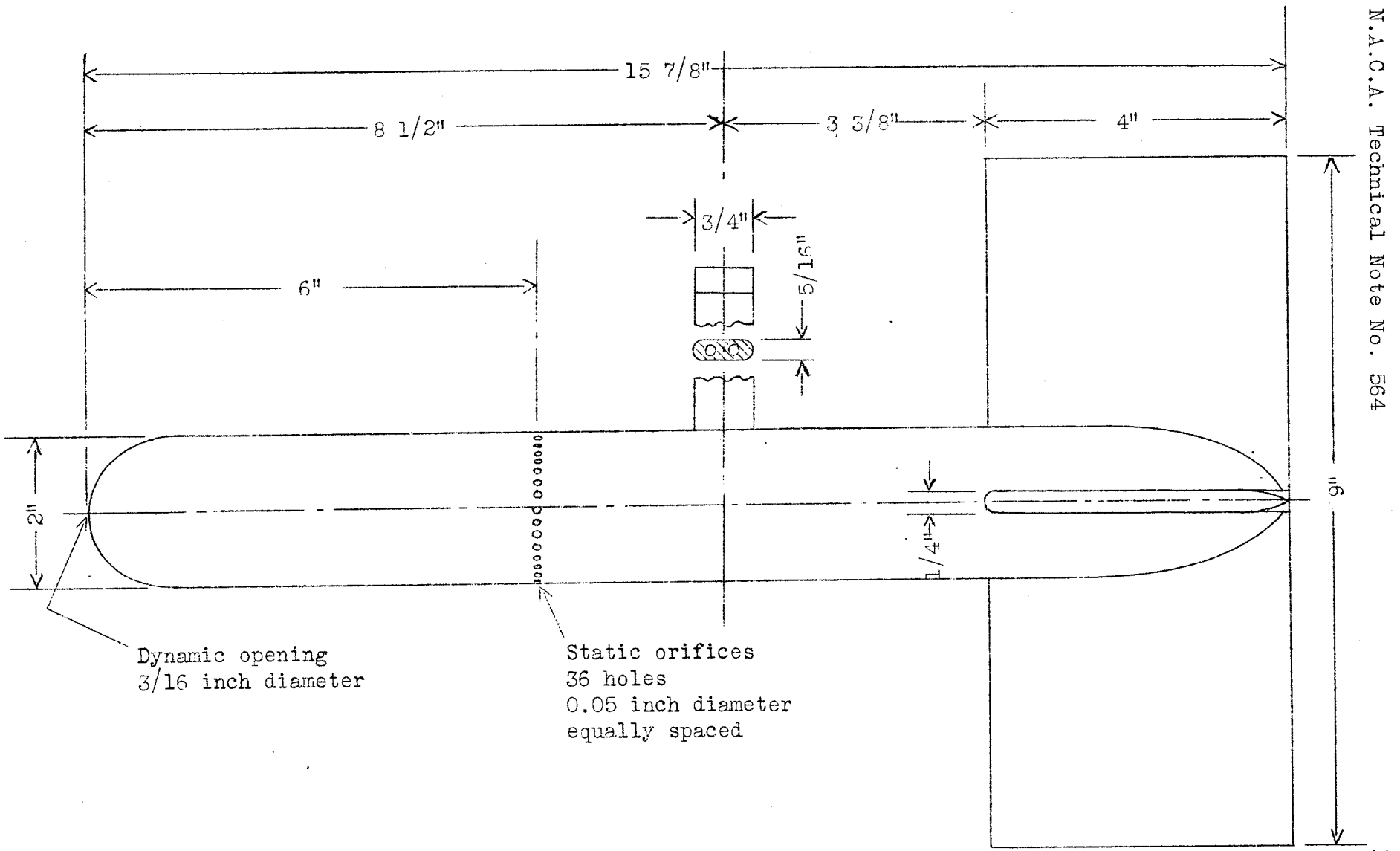


Figure 4.- Suspended air-speed head.

Fig. 4

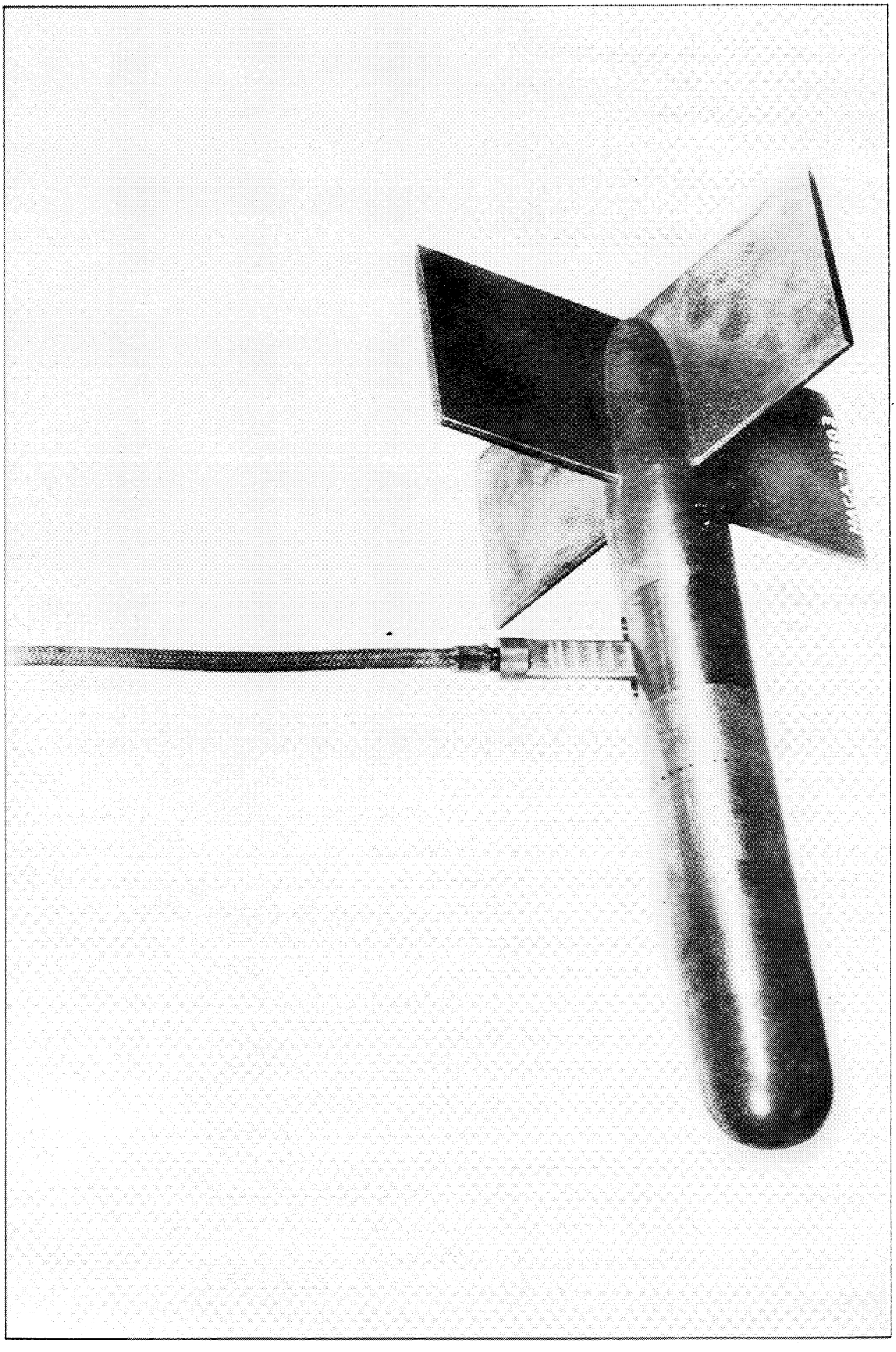
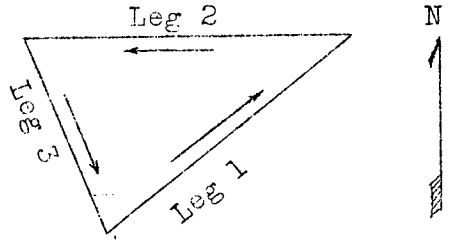


Figure 5.- Suspended air-speed head with single duct cable.



Orientation of speed course

V_1 , V_2 , V_3 , measured ground speeds
V_w ,wind speed
V,true air speed

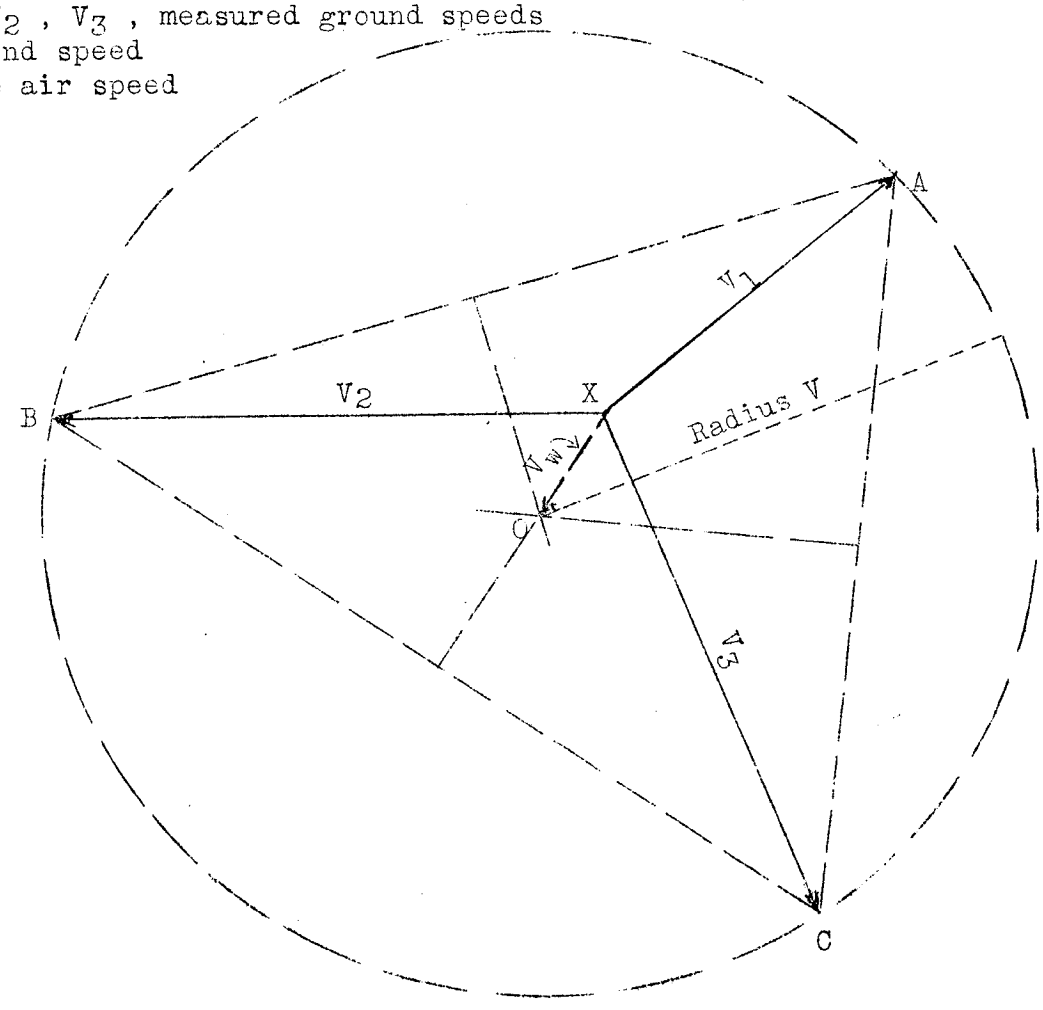


Figure 6.- Graphical method of finding true air speed from flights over a triangular speed course.

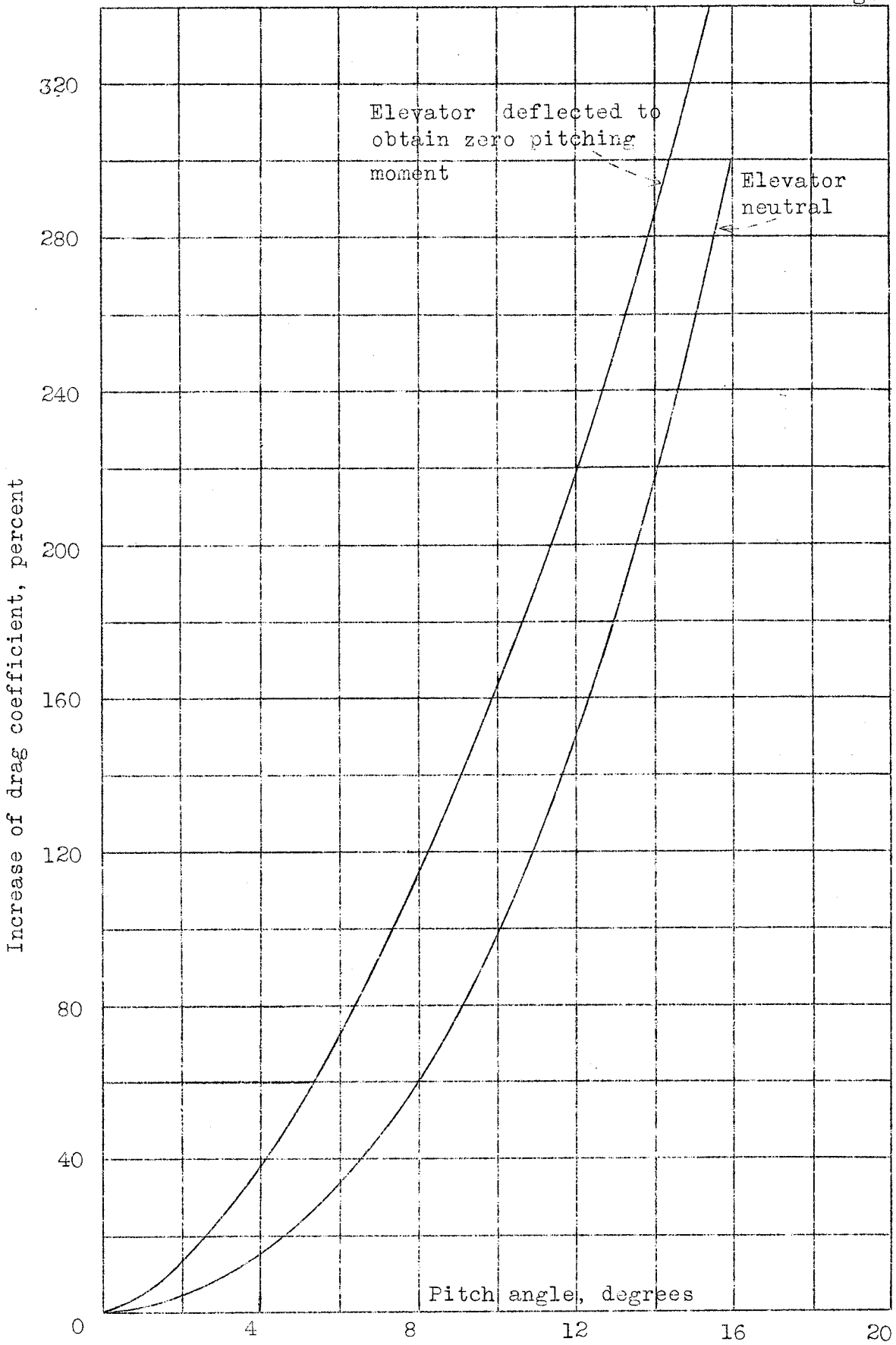


Figure 7.- Increase in drag coefficient due to pitch as a percentage of the minimum drag coefficient for U.S.S. Akron model.

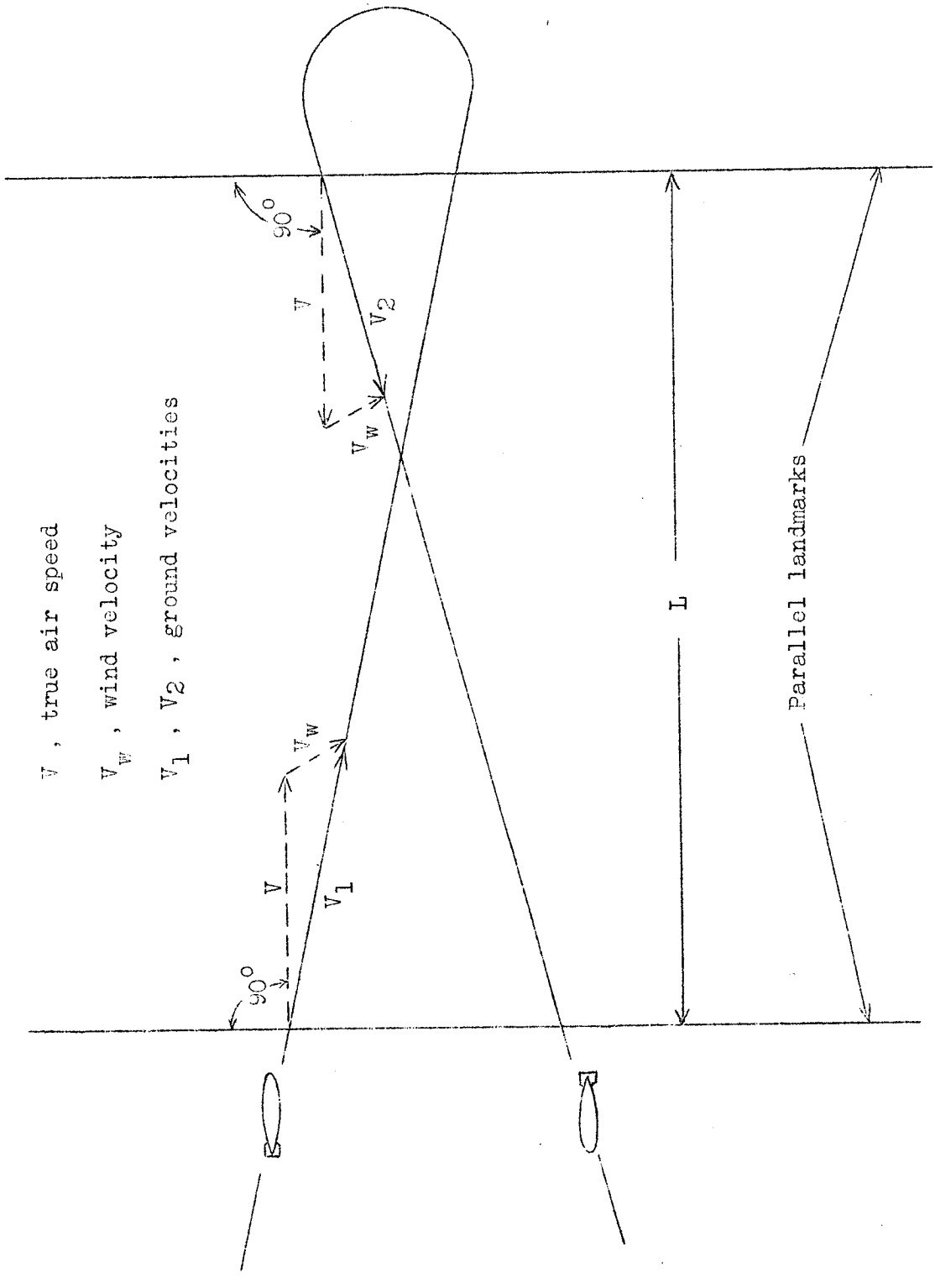


Figure 8.- Diagram of speed course with parallel landmarks.